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DEPARTMENT OF ENGINEERING

TECHNICAL STUDIES IN CARGO HANDLING - I

FORMULATION OF RECURRENCE EQUATIONS FOR SHUTTLE PROCESS AND ASSEMBLY LINE

FC BAC

Richard Bellman

REPORT 56-53

UNIVERSITY OF CALIFORNIA, LOS ANGELES

REPORT. NO. 56-53

TECHNICAL STUDIES IN CARGO HANDLING - I

Formulation of Recurrence Equations

For

Shuttle Process and Assembly Line

by

Richard Bellman

FOREWORD

The series of reports which is entitled "Technical Studies in Cargo Handling" is primarily a working paper reporting on the progress of research or the completion of a portion of a larger investigation. This study is being published in a tentative form in order to disseminate the information as quickly as possible among the several groups who are currently working on related problems. This paper may be expanded, modified, withdrawn, or published as a report in the series entitled "An Engineering Analysis of Cargo Handling" or some other form at a later date.

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INTRODUCTION

This is the first of a series of reports devoted to the study of processes which in different versions have been variously called "assembly line", "queuing," "waiting line," "bottleneck," or "scheduling processes". These may all be subsumed under the inclusive study of the flow of men and materials in space and time.

At the expense of the usual artificial discontinuity introduced by classification, the overall study, of which this paper represents the first part, may be divided into four parts:

1. Formulative
2. Computational
3. Analytic
4. Economic

It is clear, to begin with, that quantitative studies require quantitative models, which implies, inexorably, a mathematical model of the physical system we are analyzing. Of great importance is a systematic study of the types of mathematical systems which arise from the study of flow processes. As a start on this general program, we shall discuss here two specific processes, one arising in cargo-handling, a "shuttle" process, and the other arising in industrial production. Although the processes bear some structural similarity, the basic connection is by way of similarity of mathematical treatment. In subsequent reports, we shall extend this treatment to the discussion of other flow processes.

In the classification of processes, two principal types may be distinquished immediately, those of <u>deterministic</u> type and those of <u>stochastic</u> type. A process will be called deterministic if the outcome of any particular decision or action involved in the process is completely determined by the choice of the decision or action. If, however, a choice of decision or action determines a distribution of outcomes, the process will be said to be stochastic. Which process more adequately represents real, and which is an approximation, is a matter of individual taste and philosophy, usually pragmatic.

We shall employ a mathematical technique, the method of recurrence relations, which we shall demonstrate to be equally useful for the treatment of both deterministic and stochastic processes. For deterministic processes, the recurrence relations permit a simple computation of various significant quantities associated with the process, such as delay times at various stages, either by hand or by machine. Occasionally they yield explicit analytic expressions for these quantities. If the process is stochastic, the recurrence relations allow us to calculate the distribution of these quantities by various sampling techniques. The explicit relations we obtain enable us to replace simulation methods by Monte Carlo methods, thereby increasing the effectiveness of computing machines with limited capacities. Furthermore, the analytic expressions focus the spotlight of our attention upon the combinations of parameters which are most meaningful.

Finally we should like to point out that the recurrence technique is applied in identical fashion to both deterministic and stochastic versions, in its initial phase.

In this report we shall confine ourselves to an application of this technique to two mathematical models, the "link-node" model of cargo-handling, discussed in (1), (2), and the assembly line model discussed in (3), (4), (5), (6). Our object to derive the basic recurrence relations which may be used to describe the process.

In subsequent reports, we shall use these relations to study the computational and analytic aspects of the processes. These two parts of the study, combined with the contents of this paper, constitute the descriptive part of our program, wherein we are concerned with the question of determining the behavior of a particular system. Once we have answered this question, to some degree of satisfaction, we are then ready to operate on the next level of the hierarchy of problems, that of devising optimal systems, or that of improving existing systems.

Using the results obtained from the first three parts of the study, this problem will be discussed under the heading "economics" in the concluding report.

We would like to thank Y. Fukuda and M. Pollack, associated with the Department of Engineering of UCLA, for their careful reading of this paper, and for their many helpful corrections and comments.

I. THREE-STAGE SHUTTLE PROCESS

1. Formulation

Let us begin with a description of a three-stage shuttle process. There are four fixed positions, A, B, C, and D, with individuals X, Y, and Z stationed at A, B, and C respectively. The function of X is to carry an item from A to B and deliver it to Y there. X cannot return to A until Y has accepted the item at B. Upon receiving the item from X, Y conveys it to C where it is delivered to C. Y cannot return to B until Z has accepted the item. As soon as Z receives the item, he conveys it to D, and returns to C immediately.

Given a set of items, S, with associated transport times for the various stages, we wish to determine the time required to convey the items from A to D, and the total delays incurred at B and C.

We shall obtain expressions for these quantities which can be utilized for a discussion of both deterministic and stochastic processes.

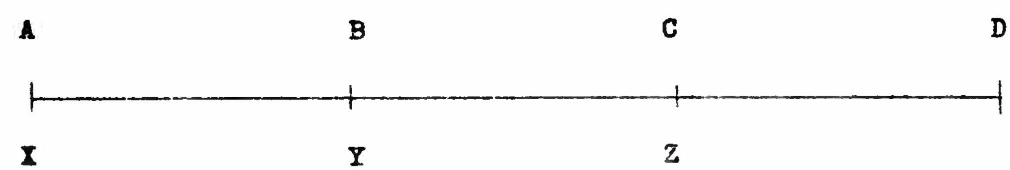


Figure 1.

Define the following quantities describing the set S:

(1) x_k = the time required for X to convey the k^{th} item from A to B

x_k' = the time required for X to return from B to A after having delivered the k^{th} item to Y.

y_k = the time required for Y to convey the k^{th} item from B to C.

y_k' = the time required for Y to return to B from C after having delivered the k^{th} item to Z.

z_k = the time required for Z to convey the k^{th} item from C to D.

z'_k = the time required for Z to return to C from D.

If the process is deterministic, we assume that the sequences (x_k), (x'_k), (y_k), (y'_k), (z_k), (z'_k) are known. If the process is stochastic, we assume that these are sequences of random variables with known distributions. Furthermore, we assume that these are independent random variables, which means no "fatigue", or "hereditary effects."

Let us further define:

(2) $d_k(X)$ = the delay to X incurred waiting for Y at B, when delivering the k^{th} item,

$d_k(Y)$ = the delay to Y incurred waiting to receive the k^{th} from X at B,

$\triangle_k(Y)$ = the delay to Y incurred waiting for Z at C, when delivering the k^{th} item,

$\triangle_k(Z)$ = the delay to Z incurred waiting to receive the k^{th} item from Y at C.

It is clear from the description of the process that:

(3) $d_k(X)d_k(Y) = 0,$

$\triangle_k(X)\triangle_k(Y) = 0,$

since one factor must always be zero.

Consider the following graph:

Figure 2.

This is a pictorial description of the activities of X, giving the times required by X for each activity. The integers k = 1,2,3, ..., signal the times at which X picks up the k^{th} item at A.

Similarly we have the graph:

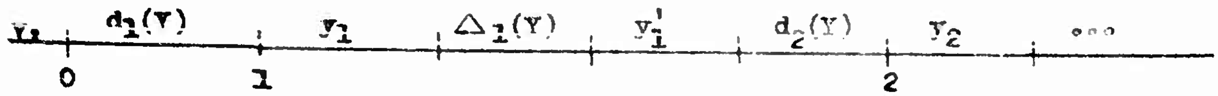


Figure 3.

This describes the activities of Y. The integers, k = 1, 2, ..., signal the times at which Y picks up the k^{th} item at B.

Finally we have the graph:

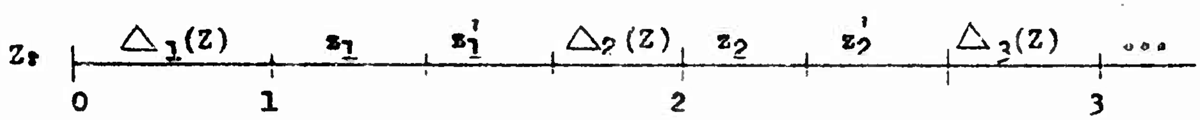


Figure 4.

where the integers signal the times at which Z picks up the k^{th} item at C.

Finally, let us now define the additional quantities:

(4) r_k = the time that X arrives at B, carrying the k^{th} item,

s_k = the time that Y arrives at B, ready to receive the k^{th} item,

t_k = the time that Y arrives at C, carrying the k^{th} item,

u_k = the time that Z arrives at C, ready to receive the k^{th} item.

2. Basic Recurrence Relations

Referring to the above time scales, we have the following relations connecting the arrival times for the $(k+1)^{st}$ item with the arrival times for the k^{th}.

$$
\begin{aligned}
(2)\quad r_{k+1} &= r_k + d_k(X) + x_k' + x_{k+1},\\
s_{k+1} &= s_k + d_k(Y) + y_k' + y_k + \triangle_k(Y),\\
t_{k+1} &= t_k + \triangle_k(Y) + y_k' + y_{k+1} + d_{k+1}(Y),\\
u_{k+1} &= u_k + \triangle_k(Z) + z_k + z_k'.
\end{aligned}
$$

Let us now express the delays, $d_k(X)$, $d_k(Y)$, $\triangle_k(Y)$, $\triangle_k(Z)$, in terms of these times of arrival. We have:

$$
\begin{aligned}
(3)\quad d_k(X) &= \text{Max}\,(s_k - r_k,\ 0),\\
d_k(Y) &= \text{Max}\,(r_k - s_k,\ 0),
\end{aligned}
$$

$$\triangle_k(Y) = \mathrm{Max}\,(u_k - t_k, 0),$$
$$\triangle_k(Z) = \mathrm{Max}\,(t_k - u_k, 0).$$

3. A Simple Observation

A result which greatly simplifies our subsequent results is the following;

$$(1)\quad d_k(X) - d_k(Y) = \mathrm{Max}\,(s_k - r_k, 0) - \mathrm{Max}\,(r_k - s_k, 0)$$
$$= s_k - r_k,$$

with the corresponding result for $\triangle_k(Y) - \triangle_k(Z)$,

$$(2)\quad \triangle_k(Y) - \triangle_k(Z) = u_k - t_k.$$

4. Explicit Recurrence Relations

Let us now combine the results of the preceding two sections so as to obtain simple recurrence relations connecting $\triangle_k(Y)$ with $\triangle_{k-1}(Y)$, and $d_k(Y)$ with $d_{k-1}(Y)$.

Combining (2.2) and (3.2), we have:

$$(1)\quad u_{k+1} - t_{k+1} = u_k - t_k + \triangle_k(Z) - \triangle_k(Y)$$
$$+ (z_k + z_k' - y_k' - y_{k+1}) - d_{k+1}(Y)$$
$$= (z_k + z_k' - y_k' - y_{k+1}) - d_{k+1}(Y).$$

Hence, referring to (2.3),

$$(2)\quad \triangle_{k+1}(Y) = \mathrm{Max}\,(u_{k+1} - t_{k+1}, 0)$$
$$= \mathrm{Max}\left[(z_k + z_k' - y_k' - y_{k+1}) - d_{k+1}(Y), 0\right].$$

Similarly,

$$(3)\quad s_{k+1} - r_{k+1} = s_k - r_k + d_k(Y) - d_k(X) + (y_k + y_k' - (x_k' + x_{k+1}))$$
$$+ \triangle_k(Y)$$
$$= (y_k + y_k' - (x_k' + x_{k+1})) + \triangle_k(Y)$$

Hence,

$$(4)\quad d_{k+1}(Y) = \mathrm{Max}\,(r_{k+1} - s_{k+1}, 0)$$
$$= \mathrm{Max}\left[-\triangle_k(Y) - (y_k + y_k' - x_k' - x_{k+1}), 0\right]$$

Combining (2) and (4), we have:

$$(5)\quad \triangle_{k+1}(Y) = \mathrm{Max}\left[\alpha_k - \mathrm{Max}\,(-\triangle_k(Y) + \beta_k, 0), 0\right],$$

where we have set for simplicity of notation,

$$(6) \quad \alpha_k = z_k + z'_k - y'_k - y_{k+1}$$

$$\beta_k = y_k + y'_k - x'_k - x_{k+1}$$

We can simplify (5) slightly,

$$(7) \quad \Delta_{k+1}(Y) = \mathrm{Max}\,(\alpha_k + \Delta_k(Y) - \mathrm{Max}\,(\Delta_k(Y), -\beta_k), 0),$$

which can also be written

$$(8) \quad \Delta_{k+1}(Y) = \mathrm{Max}\,(\Delta_k(Y), -\beta_k, \alpha_k + \Delta_k(Y)) - \mathrm{Max}(\Delta_k(Y), -\beta_k).$$

This is the fundamental recurrence relation, a basis for either a Monte Carlo treatment, or an analytic discussion.

Referring to (4),

$$(9) \quad d_{k+1}(Y) = \mathrm{Max}\Big[- \mathrm{Max}\,(z_{k-1} + z'_{k-1} - y'_{k-1} - y_k - d_k(Y), 0) - (y_k + y'_k - x'_k - x_{k+1}), 0\Big]$$

This can be written

$$(10) \quad d_{k+1}(Y) = \mathrm{Max}\Big[(-z_{k-1} - z'_{k-1} + y'_{k-1} + d_k(Y) - y'_k + x'_k + x_{k+1}), 0, -z_{k-1} - z'_{k-1} + y'_{k-1} + y_k + d_k(Y)\Big] - \mathrm{Max}\Big[-z_{k-1} - z'_{k-1} + y'_{k-1} + y_k + d_k(Y), 0\Big]$$

or, referring to (6),

$$(11) \quad d_{k+1}(Y) = \mathrm{Max}\Big[-\alpha_{k-1} + d_k(Y) - \beta_k, 0, -\alpha_{k-1} + d_k(Y)\Big] - \mathrm{Max}\Big[-\alpha_{k-1} + d_k(Y), 0\Big]$$

or

$$(12) \quad d_{k+1}(Y) = \mathrm{Max}\Big[-\beta_k, \alpha_{k-1} - d_k(Y), 0\Big] - \mathrm{Max}\Big[\alpha_{k-1} - d_k(Y), 0\Big]$$

This is also a fundamental recurrence relation.

II. THE GENERAL N-STAGE SHUTTLE PROCESS

1. Formulation

Although the above notation was satisfactory in describing a three-stage process, if we wish to describe a general N-stage process, we must employ a better notation. Consequently, we shall begin all over again, introducing a new notation.

Figure 1.

An item arriving from 0 is picked up at P_1 by X_1 who conveys it to P_2 and waits until X_2 picks it up. X_1 then returns to P_1 to await the next item. The process is repeated all down the line, until P_N; where X_N transports the item to T, the terminal.

Let

(1) $x_k(i)$ = the time required by X_k to convey the $i^{\underline{th}}$ item from P_k to P_{k+1} after having received it from X_{k-1}.

$x_k'(i)$ = the time required by X_k to return to P_k from P_{k+1} after having conveyed the $i^{\underline{th}}$ item to X_{k+1}

and let us define the delays

(2) $d_k(i)$ = the delay encountered by X_k awaiting the $i^{\underline{th}}$ item at P_k

$\triangle_k(i)$ = the delay encountered by X_k waiting to deliver the $i^{\underline{th}}$ item at P_{k+1}

for $k = 1, 2, \ldots, N$

Finally, we require the times of arrival,

(3) $r_k(i)$ = the time of arrival of X_k at P_k, ready to receive the $i^{\underline{th}}$ item,

$s_k(i)$ = the time of arrival of X_k at P_{k+1}, ready to deliver the $i^{\underline{th}}$ item.

2. Connection Between Delay Times and Arrival Times

We have the following relations connecting the above quantities

(1) $d_k(i) = \text{Max}\,(s_{k-1}(i) - r_k(i)\;,\;0),$ $\qquad k = 1, 2, \ldots$

$\triangle_k(i) = \text{Max}\,(r_{k+1}(i) - s_k(i)\;,\;0),$ $\qquad s_0(i) = 0$

from these, we conclude that

(2) $d_k(i) - \triangle_{k-1}(i) = s_{k-1}(i) - r_k(i)$

3. Basic Recurrence Relations

Let us now write down the equations connecting $r_k(i+1)$ and $s_k(i+1)$ with $r_k(i)$, $s_k(i)$, and the delays.

We have

(1) $r_k(i+1) = r_k(i) + d_k(i) + x_k(i) + x_k'(i) + \triangle_k(i),$

$s_k(i+1) = s_k(i) + \triangle_k(i) + x_k'(i) + d_k(i+1) + x_k(i+1).$

Thus

(2) $r_k(i+1) - s_{k-1}(i+1) = \triangle_k(i) - d_{k-1}(i+1)$

$+ x_k(i) + x_k'(i) - x_{k-1}'(i) - x_{k-1}(i+1).$

Hence

(3) $\triangle_{k-1}(i+1) = \text{Max}\left[\triangle_k(i) - d_{k-1}(i+1) + \alpha_k(i), 0\right],$

where

(4) $\alpha_k(i) = x_k(i) + x_k'(i) - x_{k-1}'(i) - x_{k-1}(i+1).$

Similarly, since

$$(5) \quad s_{k-1}(i) - r_k(i) = d_{k-1}(i) - \Delta_k(i-1) + x'_{k-1}(i-1) + x_{k-1}(i) - x_k(i-1) - x'_k(i-1),$$

we have

$$(6) \quad d_k(i) = \text{Max}\left[\beta_k(i) + d_{k-1}(i) - \Delta_k(i-1),\ 0\right],$$

where

$$(7) \quad \beta_k(i) = x'_{k-1}(i-1) + x_{k-1}(i) - x_k(i-1) - x'_k(i-1).$$

III. GENERAL SHUTTLE PROCESS

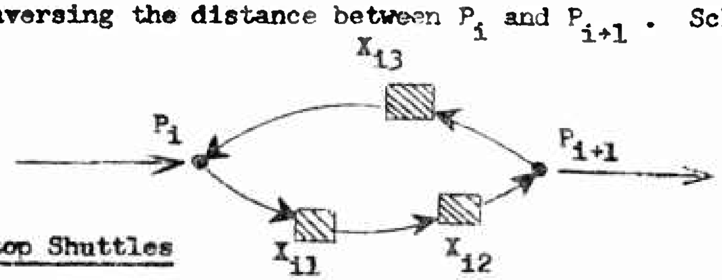
1. Storage

Let us now mention in passing a number of modifications of the above process which it may be desirable to introduce.

The first of these involves storage. Instead of assuming that X_i is required to hand each item over to X_{i+1} at P_{i+1} before returning to his position at P_i, let us assume that there are storage facilities at P_{i+1} allowing a number of items up to k_{i+1} to be stored there until X_{i+1} can transport them. This is a very important modification since much of the delay can be avoided with suitable storage capacity.

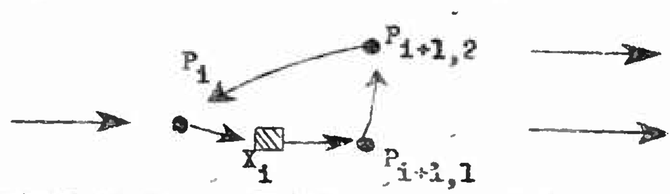
2. Multiple Shuttles

The process treated in the preceding section considered only one conveyor going back and forth between each position. A more general situation involves m_i shuttles traversing the distance between P_i and P_{i+1}. Schematically



3. Multi - Stop Shuttles

In some situations, the same shuttle must make a number of stops before returning to its original position. Schematically



The shuttle, X_i, stops at $P_{i+1,1}$ and $P_{i+1,2}$, in this order, before returning to P_i.

IV. TWO - STAGE ASSEMBLY LINE

1. Formulation

Let us now turn to the consideration of a flow process of a different type. We shall discuss a simple mathematical model of a two - stage assembly line. As will be seen from the description, the model is equally useful in the consideration of a number of other waiting-line process.

Let us suppose that we have two machines, M_1 and M_2, performing different functions, and a number of items, N, which require processing on these machines. Let

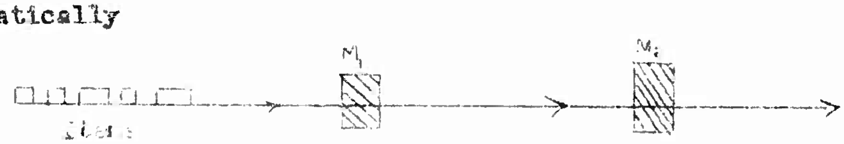
(1) x_i = the time required by the i^{th} item on the first machine,

y_i = the time required by the i^{th} item on the second machine,

and assume that each item must pass thru the first machine before going thru the second machine.

The process is carried through in the following way. The first item is fed into M_1, requiring a time x_1, and then sent to M_2, requiring a time y_1 on this machine. As soon as the first item has been processed by M_1, the second item is fed in. As soon as the second item has passed thru this first stage, it is sent to M_2. Here there may or may not be a delay, depending upon whether M_2 has completed the processing of the first item or not. The process continues in this way.

Schematically



The problem is to determine the total delay incurred in the processing of the N items, in the deterministic case, and the distribution of the delay times if the process is stochastic.

2. Recurrence Relations

Define the following quantities

(1) d_k = the delay incurred by the k^{th} item waiting upon the second machine,

r_k = the time of arrival of the k^{th} item at the waiting line for the second machine.

The basic recurrence relations are

$$(2)\quad d_{n+1} = \text{Max}\left[d_n + y_n - x_{n+1},\ 0\right],$$

$$r_{n+1} = r_n + x_{n+1}.$$

Using (2), an interesting explicit formula for d_N can be derived. We have

$$(3)\quad d_1 = 0,$$

$$d_2 = \text{Max}\left[y_1 - x_2,\ 0\right],$$

$$d_3 = \text{Max}\left[\text{Max}(y_1 - x_2,\ 0) + y_2 - x_3,\ 0\right]$$

$$= \text{Max}\left[y_1 + y_2 - x_2 - x_3,\ y_2 - x_3,\ 0\right],$$

whence it follows inductively that

$$(4)\quad d_N = \text{Max}\left[\max_{1 \le k \le N-1}\left[\sum_{i=k}^{N-1} y_i - \sum_{i=k+1}^{N} x_i\right],\ 0\right]$$

The result is due to S. Johnson, (4).

3. Optimal Arrangement

In the deterministic case, an interesting problem is that of determining the order in which the items should be fed into the machines so as to minimize the total time required by the process. This problem was resolved by S. Johnson, (4), using the explicit expression given in (2.4), and by the author, (3), using the functional equation technique of dynamic programming. In this latter paper, continuous versions of the process are also discussed.

Although the two-stage process can be treated in a number of ways, the three stage process, and general N-stage process, so far have resolutely defied either an analytic or computational approach.

1. **Formulation**

Let us now consider a corresponding three - stage process.

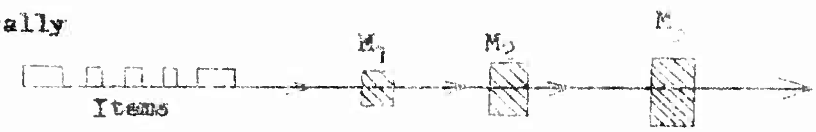
Let

(1) x_i = the time required by the i^{th} item on M_1 ,

y_i = the time required by the i^{th} item on M_2 ,

z_i = the time required by the i^{th} item on M_3 ,

Schematically



Further, let

d_i = the delay incurred by the i^{th} item waiting on the second machine,

Δ_i = the delay incurred by the i^{th} item waiting on the third machine.

r_i = the time of arrival of the i^{th} item at the waiting line for the second machine

s_i = the time of arrival of the i^{th} item at the waiting line for the third machine

These last two times are times of arrival. The times at which the items are processed are the arrival times plus the delay times.

2. **Recurrence Relations**

The basic recurrence relations are

(1) $r_k = \sum_{i=1}^{k} x_i$,

$s_k = r_k + d_k + y_k$,

$d_k = \text{Max}\left[- r_k + (r_{k-1} + d_{k-1} + y_{k-1}), 0 \right]$,

$\Delta_k = \text{Max}\left[- s_k + (s_{k-1} + \Delta_{k-1} + z_{k-1}), 0 \right]$, These may be simplified to

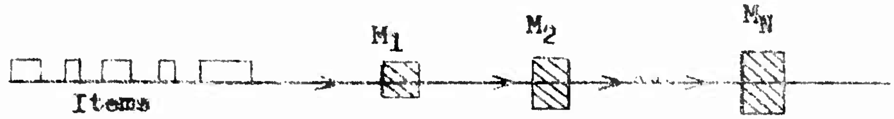
$$(2) \quad d_k = \mathrm{Max}\left[d_{k-1} + y_{k-1} - x_k \, , \, 0\right]$$

$$\triangle_k = \mathrm{Max}\left[\triangle_{k-1} + x_{k-1} + d_{k-1} + y_{k-1} - x_k - d_k - y_k \, , \, 0\right]$$

Furthermore, an explicit expression corresponding to (2.4) can be obtained, see Johnson, (4).

1. <u>Formulation</u>

As before, we must introduce a new notation to treat the N - stage process. We suppose that there are N machines, $M_1, M_2, \ldots, M_N$,



Let

(1) $x_i(k)$ = the time required by the i^{th} item on M_k, $k = 1,2,\ldots,N$, $i = 1,2,\ldots n$

$d_i(k)$ = the delay incurred by the i^{th} item waiting on the k^{th} machine

$r_i(k)$ = the time of arrival of the i^{th} item at the waiting line for the k^{th} machine.

2. <u>Recurrence Relations</u>

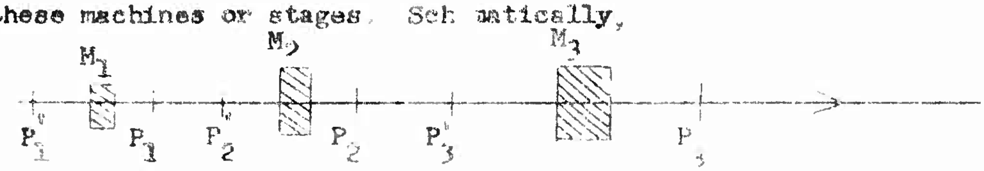
The basic recurrence relations are

(1) (a) $r_i(k) = r_i(k-1) + d_i(k-1) + x_i(k-1)$

(b) $d_i(k) = \text{Max}\left[r_{i-1}(k) + d_{i-1}(k) + x_{i-1}(k) - r_i(k),\ 0\right]$

1. Discussion

A number of flow processes combine features of both the assembly line model and the shuttle model. Consider a situation where there are a number of "machines", M_1, M_2, ..., M_N, placed in this order, and a number of items which must pass thru these machines or stages. Schematically,



The item is received at P_k' and placed on the waiting line for the machine M_k. After having gone thru the machine, it is placed on a waiting line for conveyance from P_k to P_{k+1}'. The process continues in this way.

We shall postpone any presice formulation of more general processes of this type until a later time, since our numerical and analytic pilot studies will center about the simpler models discussed in the foregoing pages.

BIBLIOGRAPHY

1. An Engineering Analysis of Cargo - Handling, Report 53-21, August 1953, Dept. of Engineering, UCLA.

2. An Engineering Analysis of Cargo - Handling, Report 55-2, November 1954, Dept. of Engineering, UCLA

3. R. Bellman and O. Gross, "Some Combinatorial Problems Arising in the Theory of Multi - Stage Process," Jour. Soc. Ind. Appl. Math., Vol. 2(1954), pp 175-183.

4. S. Johnson, "Optional Two- and Three- Stage Production Schedules with Setup Times Included," Naval Research Logistics Quarterly. March, 1954.

5. M. Salveson, "On a Qualitative Method in Production Planning and Scheduling," Econometrics, Vol. 20(1952), pp 571-4.

6. A Vasonyi, "The Use of Mathematics in Production and Inventory Control," Management Science, Vol. 1(1954), pp 70-85.

An excellient discussion of queuing theory, together with an extensive bibliography, may be found in

D. G. Kendall, "Some Problems in the Theory of Queues," Jour. Royal Stat. Soc, Ser B, Vol. XIII, 1951, pp 151-185.

A general discussion of mathematical problems arriving in flow processes may be found in

R. Bellman, "Mathematical Aspects of Scheduling Theory," Jour. Soc. Ind. Appl. Math.,(to appear),

where a brief bibliography of this field may be found.